Bdot Controller Code
Official Release v2

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IMPORTANT NOTE: The equations given in this document use the units defined in the nomenclature. They will need to be adapted to whatever are useful units.

1 Nomenclature

Symbol	Description	Type	Status
$\vec{B}$	Magnetic field measurement / reference [T]	[-]	[-]
B_i	Components of magnetic field [T]	int16[1]	V
K_B	Inverse magnitude of magnetic field squared [1/T^2]	intern	V
$\dot{\vec{B}}$	Calculatd derivative of magnetic field [T/s]	intern	V
$Bdot_i$	Components of magnetic field derivative [T/s]	intern	V
$\vec{I}$	Current to magnetotorquers [A]	[-]	[-]
I_i	Components of current to magnetotorquer [A]	int16[2]	V
I_{limit}	Maximum current allowed through the coils [A]	intern	C
I_{max}	Maxmimum requested current, [A]	intern	V
A	Area surrounded by magnetotorquer coils [m^2]	intern	C
n	Number of windings per magnetotorquer[-]	intern	C
K_{Bdot}	Controller gain [Nms]	intern	HK
$(.)_k$	Signal at time $t = k$	[-]	[-]
$(.)_{k-1}$	Signal at time $t = k - 1$	[-]	[-]
λ	Derivative stage parameter [-]	intern	HK
Δt	Sampling time [s]	intern	HK
K_s	Sign of Bdot gain, [-], (acceleration, deceleration)	intern	V
treshold	Desired value of minimum rotational speed [deg]	intern	HK

For explanation: type referres to the data type. 'intern' means it is basically up to the one implementing the code, what data type is choosen. 'int16' means that from/to the exterieur a signed 16bit value is passed. Most information defining the interfaces and the hardware chosen for encoding measurements (thus defining the digital representation) can be found in [1]. 'Status' referres to how the variable shall be treated, *i.e.* if it is a variable with changing magnitude,

V, a variable with constant magnitude, C, or if it shall be a 'house keeping' variable, HK.

2 Value Ranges of Integers

Bits	Type	Range		
12	signed	-2048	to	2047
12	unsigned	0	to	4095
16	signed	-32768	to	32767
16	unsigned	0	to	65536
32	signed	$-2.147483648 \cdot 10^{-9}$	to	$2.147483647 \cdot 10^{9}$

3 Magnetometers Description

In [1] sensitivity is given to be 27 nT together with a 16bit ADC.

4 Governing Equations

$$\dot{\vec{B}}_k = (1-\lambda) \cdot \dot{\vec{B}}_{k-1} + \lambda \cdot \frac{\vec{B}_k - \vec{B}_{k-1}}{\Delta t} \tag{1}$$

$$\frac{\vec{\mu}_k}{An} = \vec{I}_k = \frac{1}{|\vec{B}_k|^2} \cdot K_s \cdot \dot{\vec{B}}_k \cdot K_{Bdot} \cdot \frac{1}{An} \tag{2}$$

$$K_s = \begin{cases} -1 & \text{if } |\vec{\omega}| >= \text{treshold} \\ +1 & \text{if } |\vec{\omega}| < \text{treshold} \end{cases} \tag{3}$$

5 Code to be Implemented

Inputs:

$$\begin{array}{c} B_{x_k} \\ B_{y_k} \\ B_{z_k} \end{array}$$

Equation (1) in components:

$$Bdot_{x_k} = (1-\lambda) \cdot Bdot_{x_{k-1}} + \lambda \cdot \frac{B_{x_k} - B_{x_{k-1}}}{\Delta t} \tag{4}$$

$$Bdot_{y_k} = (1-\lambda) \cdot Bdot_{y_{k-1}} + \lambda \cdot \frac{B_{y_k} - B_{y_{k-1}}}{\Delta t} \tag{5}$$

$$Bdot_{z_k} = (1-\lambda) \cdot Bdot_{z_{k-1}} + \lambda \cdot \frac{B_{z_k} - B_{z_{k-1}}}{\Delta t} \tag{6}$$

Norm of the rotational speed:

$$|\vec{\omega}| = \sqrt{{\omega_{x_k}}^2 + {\omega_{y_k}}^2 + {\omega_{z_k}}^2} \tag{7}$$

The sign of the Bdot controller is then given by

$$K_s = \begin{cases} -1, & \text{if } |\vec{\omega}| >= \text{treshold} \\ +1, & \text{if } |\vec{\omega}| < \text{treshold} \end{cases} \tag{8}$$

Equation (2) in components:

$$K_B = \frac{1}{B_{x_k}^2 + B_{y_k}^2 + B_{z_k}^2} \tag{9}$$

$$I_{x_k} = K_B \cdot Bdot_{x_k} \cdot K_s \cdot K_{Bdot} \cdot \frac{1}{An} \tag{10}$$

$$I_{y_k} = K_B \cdot Bdot_{y_k} \cdot K_s \cdot K_{Bdot} \cdot \frac{1}{An} \tag{11}$$

$$I_{z_k} = K_B \cdot Bdot_{z_k} \cdot K_s \cdot K_{Bdot} \cdot \frac{1}{An} \tag{12}$$

Note, that K_B as magnitude squared in [nT] may reach the limit of signed int32 range!

Power consumption must be limited:

$$I_{max_k} = \max\{I_{x_k}, I_{y_k}, I_{z_k}\} \tag{13}$$

If I_{max_k} is greater than a certain limit I_{limit}, all the current components must be corrected (i.e. the vector must be scaled in order to prevent the direction from changing):

$$K_i = \frac{I_{limit}}{I_{max_k}} \tag{14}$$

$$I_{x_k} = I_{x_k} \cdot K_i \tag{15}$$

$$I_{y_k} = I_{y_k} \cdot K_i \tag{16}$$

$$I_{z_k} = I_{z_k} \cdot K_i \tag{17}$$

Note, that it must be ensured, that a maximum requested current does not cause an overflow, *i.e.* I_{max_k} may be much greater than I_{limit}! See section 7 for details. Outputs:

$$I_{x_k}$$
$$I_{y_k}$$
$$I_{z_k}$$

6 Numerical Values

The following table summarizes some values, which, by simulation, have proved to work properly. Others are precisely known because they are geometric values.

Variable	Value
A	$4.861 \cdot 10^{-3}$ [m^2]
n	427 [-]
K_{Bdot}	$1.146 \cdot 10^{-4}$ [Nms]
λ	0.5 [-]

7 Maximum Requested Current

The governing equation for maximum torque and thus maximum current can – according to Appendix A – be stated as

$$|\vec{T}|_{max} = |\mu| \cdot |\vec{B}| = \dot{\vec{B}} \cdot K_{Bdot} \cdot \frac{1}{|\vec{B}|} \tag{18}$$

The maximum current is then found by solving for μ and dividing it by torquer area and number of windings.

For a given rotational speed the magnetic field derivative is proportional to the magnetic field, *i.e.* $\dot{B} \propto B$. Inserting this into the definition of μ yields a relation $\mu \propto \frac{B}{B^2} = B^{-1}$ (Note that the resulting torque T is independent of the magnetic field!). So the highest current can be expected when the magnetic field is weakest. Assuming a minimum value of 20000 nT and a rotational speed of $10°$/s with the proposed controller gain of $1.146 \cdot 10^{-4}$ Nms, this gives a current of around 240 mA. This value is around 12 higher than the maximum current allowed. Nevertheless this value must be taken into account in order to properly scale the whole current vector!

8 Contact Information

If there are any questions regarding the code, please refer to Martin Ehrensperger by email, `martin.ehrensperger@epfl.ch`.

A Bdot in Detail

The goal of Bdot control is to minimize / stabilize the rotational speed of a given spacecraft. Thus the torque $\vec{T}$ created by the controller shall oppose the current rotational speed $\vec{\omega}$, *i.e.*

$$\frac{\vec{T}}{|\vec{T}|} = -\frac{\vec{\omega}}{|\vec{\omega}|}$$

Further the torque created by a magnetic dipole in interaction with the earth magnetic field is given by

$$\vec{T} = \vec{\mu} \times \vec{B}$$

Combining these two equations yields

$$\begin{aligned}
\vec{\mu} \times \vec{B} &= \vec{T} \\
\vec{B} \times (\vec{\mu} \times \vec{B}) &= \vec{B} \times \vec{T} \\
\vec{I} \times (\vec{\mu} \times \vec{I}) &= \vec{B} \times \vec{T} \cdot \frac{1}{|\vec{B}|^2} \\
\vec{\mu} &= \vec{B} \times (-\vec{\omega}\frac{|\vec{T}|}{|\vec{\omega}|}) \cdot \frac{1}{|\vec{B}|^2} \\
\vec{\mu} &= \dot{\vec{B}} \cdot K_{Bdot} \cdot \frac{1}{|\vec{B}|^2} \qquad (19)
\end{aligned}$$

where $\vec{I}$ is a unit vector and $\vec{B} \times -\vec{\omega}$ can be identified as $\dot{\vec{B}}$. Note that the 'controller gain' K_{Bdot} is not a dimensionless parameter, but stands for torque per radiant per second, *i.e.* has a unit [Nms]. Note also that acceleration is not covered in this section.

References

[1] H. Peter-Contesse. Phase B/C, ADCS System Engineering. 2008.